Name solutions

December 1, 2011

ECE 311 Exam 3 Fall 2011

Closed Text and Notes

1) Be sure you have 9 pages.

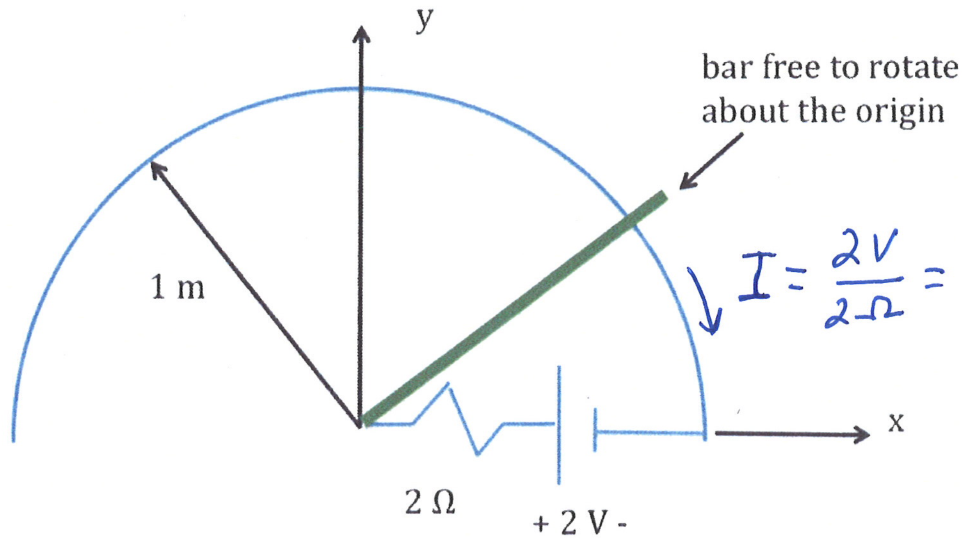
2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(15 pts) 1. A conducting bar is free to rotate about the pivot at the origin. The other end of the bar freely moves, and makes electrical contact to a semi-circular conducting rail of radius 1 m. Everywhere there is a magnetic flux density of $\mathbf{B} = -10 \frac{\text{Wb}}{\text{m}^2} \mathbf{a}_z$. What is the force on the bar?



$$I = \frac{2V}{2\Omega} = 1A$$

$$\vec{F} = \int I \, d\vec{l} \times \vec{B} = \int_0^{1m} (1A) \, d\rho \, \hat{a}_\rho \times \left(-10 \frac{Wb}{m^2} \hat{a}_z\right)$$

$$= -10 A \frac{Wb}{m^2} \int_0^{1m} d\rho \left(-\hat{a}_\phi\right)$$

$$= 10 A \frac{Wb}{m^2} \rho \Big|_0^{1m} \hat{a}_\phi$$

$$= 10 A \frac{Wb}{m^2} \hat{a}_\phi$$

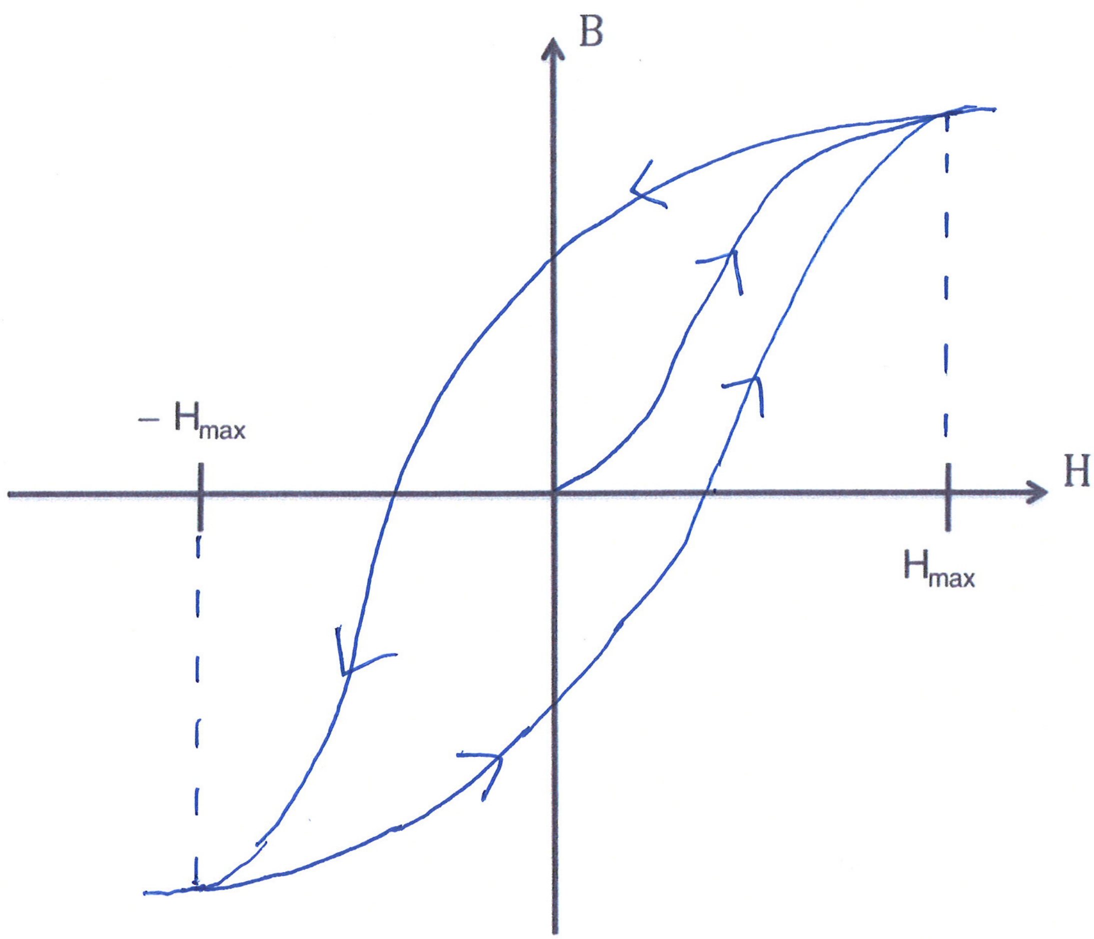
$$= 10 N \hat{a}_\phi$$

$$\left(\frac{A\,Wb}{m} = \frac{A\,V\,s}{m} = \frac{C}{s} \frac{J}{C} \frac{s}{m} = \frac{J}{m} = \frac{Nm}{m} = N \right)$$

(6 pts) 2. A current of 1 mA is flowing in a circular wire of radius 0.02 m. If the fingers of one's right hand are curled in the direction of the current in the loop, then one's thumb points in the direction $\hat{\mathbf{a}}_m$. If the loop is free to rotate, what direction would $\hat{\mathbf{a}}_m$ point if the loop is placed in a uniform magnetic flux density $\mathbf{B} = -10\hat{a}_y$ T?

$-\hat{a}_y$

(7 pts) 3. You are given a piece of iron that is demagnetized. On the graph below, sketch the B versus H behavior for this piece of iron as H is first increased from 0 to H_{max}, decreased to $-H_{max}$, and then increased back to H_{max}. Be sure to include arrows on the characteristic to indicate the direction of the characteristic.



(6 pts) 4. Two parallel wires carry currents in the opposite direction. The force experienced by one due to the other is

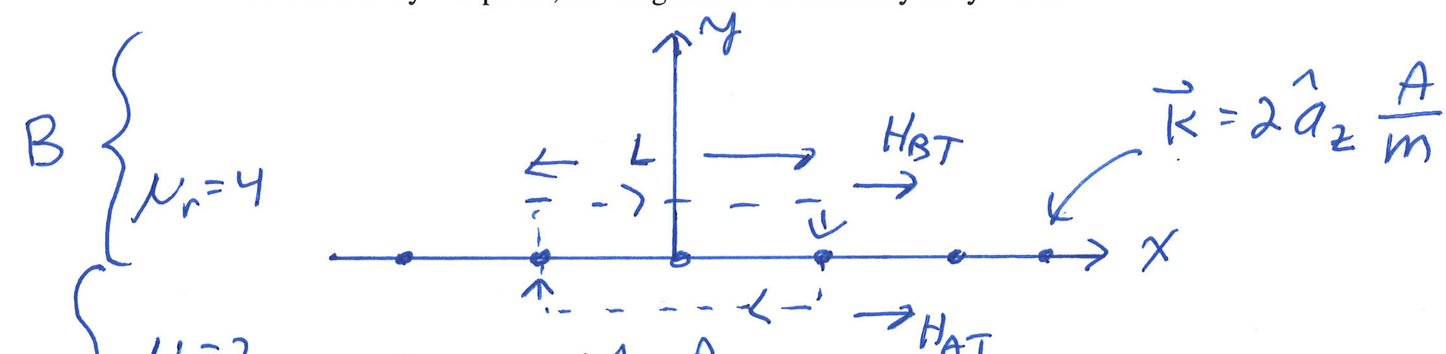
A) parallel to the wires

B) perpendicular to the wires and attractive

C) perpendicular to the wires and repulsive

D) zero.

(10 pts) 5.The region for y < 0 has relative permeability of 2 and the region for y > 0 a relative permeability of 4. If $\mathbf{H} = (6\hat{a}_x + 6\hat{a}_y)\frac{A}{m}$ for y < 0 and there is a sheet current density of $\mathbf{K} = 2\hat{a}_z\frac{A}{m}$ flowing on the in the y = 0 plane, the magnetic field intensity for y > 0 is



$$H_{An} = 6\hat{a}_y \frac{A}{m}$$

$$H_{AT} = 6\hat{a}_x \frac{A}{m}$$

$$B_{An} = 2\mu_0 H_{An} = 2\mu_0 6 = B_{Bn} = 4\mu_0 H_{Bn}$$

$$H_{BN} = \frac{2}{4} H_{AN} = \frac{2}{4} 6\hat{a}_y \frac{A}{m} = 3\hat{a}_y \frac{A}{m}$$

$$\oint \vec{H} \cdot d\vec{\ell} = H_{BT} L - H_{AT} L = -KL = \left(-2\frac{A}{m}\right) L$$

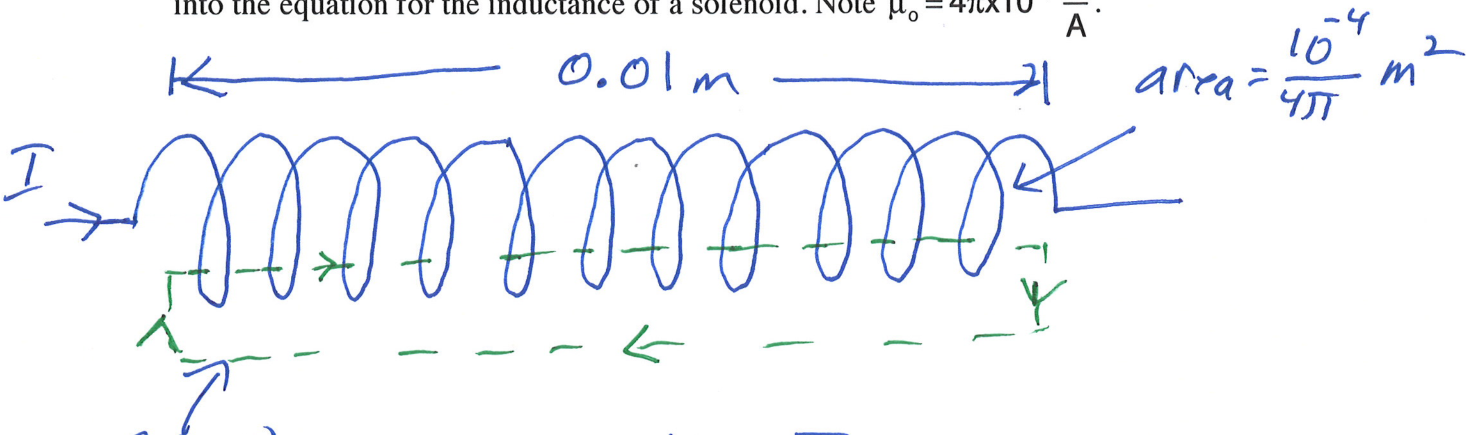
over dashed path shown

I enclosed by dashed path

$$H_{BT} = -2\frac{A}{m} + H_{AT} = -2\frac{A}{m} + 6\frac{A}{m} = 4\frac{A}{m}$$

$$\vec{H}_B = (4\hat{a}_x + 3\hat{a}_y)\frac{A}{m}$$

(15 pts) 6. A solenoid consists of 1000 turns, a length of 0.01 m, and a cross-sectional area of $\frac{10^{-4}}{4\pi}\text{m}^2$. The center of the solenoid contains a material with a relative permeability of $\mu_r = 1000$. Find the inductance of this solenoid starting from basics, do not just plug the parameters of the solenoid into the equation for the inductance of a solenoid. Note $\mu_o = 4\pi x 10^{-7}\frac{\text{H}}{\text{A}}$.



$$\oint \vec{H}\cdot d\vec{l} = I_{encl} = 1000\, I$$

$$H\,(0.01\text{m}) = 1000\, I \Rightarrow H = 10^5\, I\, m^{-1}$$

$$B = \mu_o \mu_r = \left(4\pi \times 10^{-7}\frac{H}{m}\right)(10^3)\, 10^5\, I\, m^{-1}$$

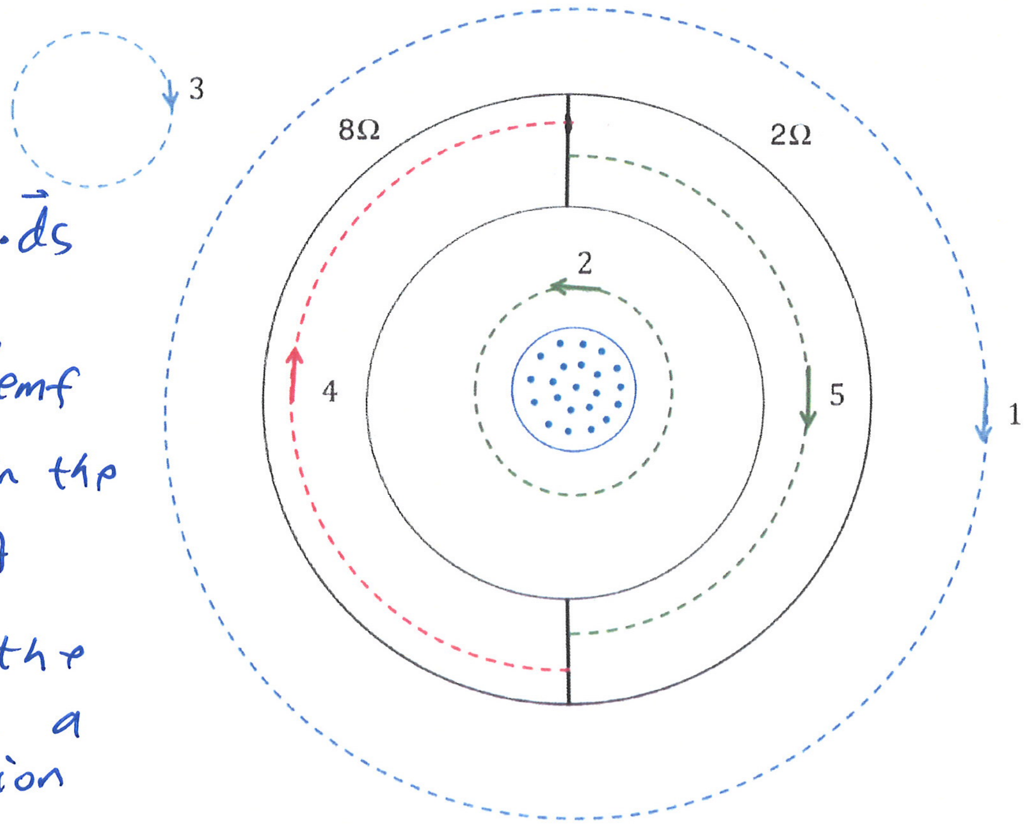
$$\phi = BA = 4\pi \times 10^1\, I\left(\frac{H}{m^2}\right)\frac{10^{-4}}{4\pi}\, m^2$$

$$\phi = 10^{-3}\, I\, H$$

$$L = \frac{\lambda}{I} = \frac{N\phi}{I} = \frac{(10^3)(10^{-3})\, I\, H}{I}$$

$$L = 1\, H$$

(15 pts) 7. A conducting ring is composed of two different materials. The half-ring on the left has a total resistance of 8Ω and the half ring on the right a total resistance of 2Ω. A solenoid goes through the ring. The dots in the center of the solenoid indicate the direct the magnetic flux is increasing due to an increasing current in the solenoid resulting in a rate of change of magnetic flux out-of-the page of $\frac{d\phi}{dt} = 10\,V$. The dashed lines indicate paths. For the paths shown find the following,



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}$$

$$= -\frac{d\varphi}{dt} = V_{emf}$$

So the current in the ring $= \frac{10V}{10\Omega} = 1\,A$

From Lenz's law the current flows in a clockwise direction

$\oint_1 \mathbf{E} \bullet \mathbf{dl} =$ 10V

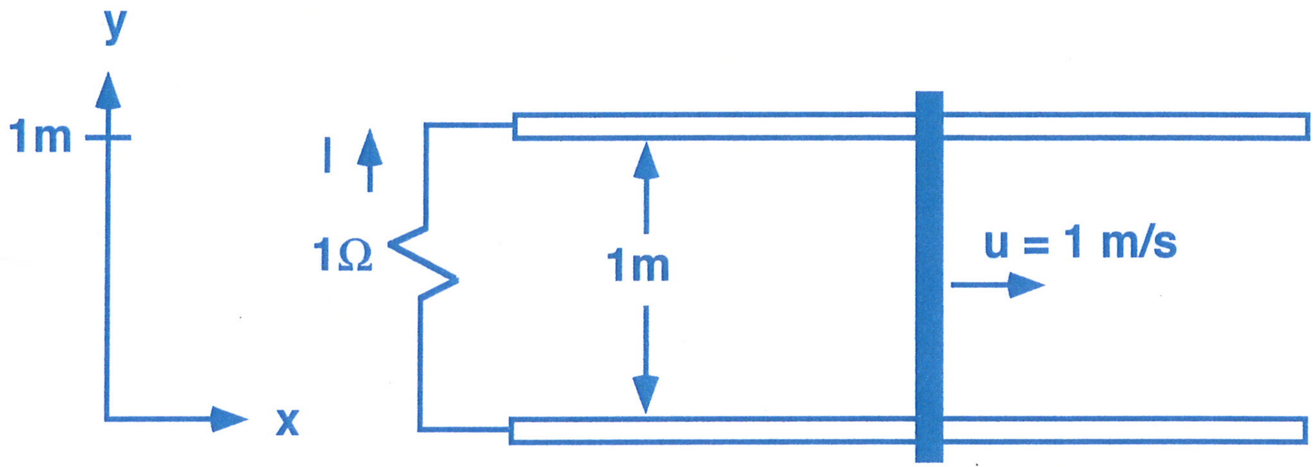
$\oint_2 \mathbf{E} \bullet \mathbf{dl} =$ −10V

$\oint_3 \mathbf{E} \bullet \mathbf{dl} =$ 0

$\int_4 \mathbf{E} \bullet \mathbf{dl} =$ 8V

$\int_5 \mathbf{E} \bullet \mathbf{dl} =$ 2V

(20 pts) 8. Two rails have negligible resistance and attached to the rails is a resistor, as shown. A 0.1 kg metal bar of negligible resistance is laid across the rails and is being pulled in the +x direction at 1 m/s. Every where is a magnetic field intensity of $\mathbf{H} = \frac{10}{\mu_o} \frac{Wb}{m^2} \mathbf{a}_z$.



(10 pts)A) What is the current I, referenced as shown, flowing through the resistor is,

$$\vec{B} = \mu_o \vec{H} = 10 \frac{Wb}{m^2} \hat{a}_z$$

$$\phi = BA = \left(10 \frac{Wb}{m^2}\right)(1m)\, x$$

$$\frac{d\phi}{dt} = 10 \frac{Wb}{m} \frac{dx}{dt} = 10 \frac{Wb}{m}(u) = \left(10 \frac{Wb}{m}\right)\left(1 \frac{m}{s}\right)$$

$$= 10 V$$

$$I = \frac{10V}{1\Omega} = 10 A$$

(10 pts) B) What force is applied to the bar so that it is moving as shown?

$$\vec{F}_m = \int I d\vec{l} \times \vec{B} = \int_{1m}^{0} (10A) dy\, \hat{a}_y \times 10 \frac{Wb}{m^2} \hat{a}_z$$

$$= 100 \frac{Wb A}{m^2} \hat{a}_x \int_{1m}^{0} dy = 100 \frac{Wb A}{m^2} (-1m)\, \hat{a}_x = -100 N \hat{a}_x$$

So $\vec{F}_{APPL} = -\vec{F}_m = 100 N \hat{a}_x$

(6 pts) 9. A parallel plate capacitor is being charged at the rate of $\frac{dQ}{dt}=1\frac{C}{s}$. If the plate area is $1m^2$ and the plate separation is 0.001 m, what is the displacement current density between the capacitor plates?

$$I_d = \frac{dQ}{dt} = 1\frac{C}{s} = 1A$$

$$J_d = \frac{I_d}{A} = \frac{1A}{1m^2} = 1\frac{A}{m^2}$$